$$\int x \ln(1+x^2)\,dx$$

微 $\downarrow$ $\dfrac{u=\ln(1+x^2)}{du=\dfrac{2x}{1+x^2}dx}$ $\quad$ $\dfrac{dv = x\,dx}{v=\frac{1}{2}x^2}$ $\downarrow$ 積.

$$\overset{u\cdot v}{=\frac{1}{2}x^2\cdot\ln(1+x^2)} - \frac{1}{2}\int x^2\cdot\overset{v\cdot du}{\frac{2x}{1+x^2}}\,dx$$

$$=\frac{1}{2}x^2\cdot\ln(1+x^2)-\frac{1}{2}\int\frac{2x^3}{1+x^2}\,dx$$

→ 假分式，分子>分母.

① 長除法

② 綜合除法

$$\begin{array}{r|l} 1+0+1 & \overline{2+0} \quad \text{商} \\ & 2+0+0+0 \\ & 2+0+2 \\ \hline \text{被除式} & 0-2+0. \\ & 0+0+0 \\ \hline & -2+0 \\ & \text{餘}. \end{array}$$

$$=\frac{1}{2}x^2\cdot\ln(1+x^2)-\frac{1}{2}\int 2x+\frac{-2x}{1+x^2}\,dx$$

$$=\frac{1}{2}x^2\cdot\ln(1+x^2)-\frac{1}{2}\int 2x-\frac{2x}{1+x^2}\,dx$$

$$=\frac{1}{2}x^2\cdot\ln(1+x^2)-\frac{1}{2}\left(x^2-\ln|1+x^2|\right)+C \qquad \int\frac{f'(x)}{f(x)}\,dx$$

$$=\frac{1}{2}x^2\cdot\ln(1+x^2)-\frac{1}{2}x^2+\frac{1}{2}\ln|1+x^2|+C \qquad = \ln|f(x)|+C$$

$$\frac{1}{2}x^2\ln(1+x^2)-\frac{1}{2}x^2+\frac{1}{2}\ln|1+x^2|+C \;\#$$