

$$(a) \hat{\beta}_1 = \frac{S_{XY}}{S_{XX}} = \frac{\sum_{i=1}^n X_i Y_i - \frac{(\sum_{i=1}^n X_i)(\sum_{i=1}^n Y_i)}{n}}{\sum_{i=1}^n X_i^2 - \frac{(\sum_{i=1}^n X_i)^2}{n}}$$

$$= \frac{55570 - \frac{(714)(766)}{10}}{52366 - \frac{714^2}{10}} = 0.633$$

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} = 76.6 - 0.633 \times 71.4 = 31.4$$

$$\Rightarrow \text{方程為 } \hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i = 31.4 + 0.633 X_i \#$$

X_i	Y_i	X_i^2	Y_i^2	$X_i Y_i$	
70	83	4900	6889	5810	
80	88	6400	7744	7040	
67	73	4489	5329	4891	
54	72	2916	5184	3888	
88	80	7744	6400	7040	
64	57	4096	4489	4288	
54	56	2916	3136	3024	
73	81	5329	6561	5913	
90	87	8100	7569	7830	
74	79	5476	6241	5846	
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Total	714	766	52366	59542	55570

(b). $H_0 = \beta_1 = 0$

$H_1 = \beta_1 \neq 0$

拒絕 H_0 if $T > t_{\alpha/2}(n-2)$.

$$T = \left| \frac{r \sqrt{n-2}}{\sqrt{1-r^2}} \right| = \left| \frac{0.801 \times \sqrt{8}}{\sqrt{1-0.801^2}} \right| = 3.78$$

$$t_{\alpha/2}(n-2) = t_{0.025}(8) = 2.306$$

從 $T = 3.78 > t_{\alpha/2}(n-2) = 2.306$, 在 $\alpha = 0.05$ 之下拒絕 H_0 , 具線性相關。

$$r = \frac{S_{XY}}{\sqrt{S_{XX} S_{YY}}} = \frac{55570 - \frac{(714)(766)}{10}}{\sqrt{52366 - \frac{714^2}{10}} \sqrt{59542 - \frac{766^2}{10}}} = 0.801$$

(c) σ^2 估計值 = $MSE = \frac{SSE}{n-2} = \frac{S_{YY} - \hat{\beta}_1^2 S_{XX}}{n-2} = \frac{866.7 - 0.633^2 \times 1386.7}{8} = 38.861 \#$

(d) $\left[\hat{\beta}_1 - t_{\alpha/2}(n-2) \sqrt{\frac{MSE}{S_{XX}}}, \hat{\beta}_1 + t_{\alpha/2}(n-2) \sqrt{\frac{MSE}{S_{XX}}} \right]$

$$= \left[0.633 - 2.306 \sqrt{\frac{38.861}{1386.7}}, 0.633 + 2.306 \sqrt{\frac{38.861}{1386.7}} \right]$$

$= [0.248, 1.018]$ 為 β_1 之 95% C.I. #