

(a) 相關係數 $r = \frac{S_{XY}}{\sqrt{S_{XX}} \sqrt{S_{YY}}} = \frac{\sum_{i=1}^n X_i Y_i - \frac{(\sum_{i=1}^n X_i)(\sum_{i=1}^n Y_i)}{n}}{\sqrt{\left(\sum_{i=1}^n X_i^2 - \frac{(\sum_{i=1}^n X_i)^2}{n}\right) \left(\sum_{i=1}^n Y_i^2 - \frac{(\sum_{i=1}^n Y_i)^2}{n}\right)}}$

$$= \frac{78800 - \frac{600 \times 1200}{10}}{\sqrt{39400 - \frac{600^2}{10}} \sqrt{157660 - \frac{1200^2}{10}}} = 0.9978.$$

X_i	Y_i	X_i^2	Y_i^2	$X_i Y_i$
40	83	1600	6889	3320
30	60	900	3600	1800
70	138	4900	19044	9660
90	180	8100	32400	16200
50	97	2500	9409	4850
60	118	3600	13924	7080
70	145	4900	21025	10150
40	79	1600	6241	3160
80	158	6400	24964	12640
70	142	4900	20164	9940

Total | 600 1200 39400 157660 78800.

($r > 0$, 表正相關)

(b) $H_0 = \beta_1 = 0$
 $H_1 = \beta_1 \neq 0$

拒絕 H_0 若 $|T| > t_{\alpha/2}(n-2)$.

$$|T| = \left| \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} \right| = \left| \frac{0.9978\sqrt{10-2}}{\sqrt{1-0.9978^2}} \right| = \left| \frac{2.822}{0.1066} \right| = 42.75.$$

$$t_{\alpha/2}(n-2) = t_{0.025}(8) = 2.306.$$

從 $|T| > t_{\alpha/2}(n-2)$, 在 $\alpha = 0.05$ 下, 拒絕 $H_0 \Rightarrow$ 有線性相關

(c) $\hat{\beta}_1 = \frac{S_{XY}}{S_{XX}} = \frac{\sum_{i=1}^n X_i Y_i - \frac{(\sum_{i=1}^n X_i)(\sum_{i=1}^n Y_i)}{n}}{\sum_{i=1}^n X_i^2 - \frac{(\sum_{i=1}^n X_i)^2}{n}} = \frac{78800 - \frac{600 \times 1200}{10}}{39400 - \frac{600^2}{10}} = 2$

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} = \frac{1200}{10} - 2 \times \frac{600}{10} = 0$$

\Rightarrow 方程式 $= \hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i = 2X_i$