

反函數的導函數

若 $y = x^3 + 2x + 7$, 求 $\frac{dx}{dy}$

$$\therefore \frac{dy}{dx} = 3x^2 + 2, \therefore \frac{dx}{dy} = \frac{1}{3x^2 + 2}$$

隱函數的導函數

若 $x^2y^3 + y^2 - 5y - x^2 = -4$, 求 $\frac{dy}{dx}$ 及 $\frac{dx}{dy}$

$$\frac{d}{dx}(x^2y^3 + y^2 - 5y - x^2) = \frac{d}{dx}(-4)$$

$$3x^2y^2 \frac{dy}{dx} + 2xy^3 + 2y \frac{dy}{dx} - 5 \frac{dy}{dx} - 2x = 0$$

$$\boxed{\frac{dy}{dx} \cdot \frac{dx}{dy} \cdot y^2}$$

$$(3x^2y^2 + 2y - 5) \frac{dy}{dx} = 2x(1 - y^3)$$

$$\frac{dy}{dx} = \frac{2x(1 - y^3)}{3x^2y^2 + 2y - 5}$$

$$\frac{dx}{dy} = \frac{3x^2y^2 + 2y - 5}{2x(1 - y^3)}$$