

$$\text{設 } x^2 - 3xy + y^2 = 1, \text{ 求 } \frac{d^2y}{dx^2}$$

$$\frac{d}{dx}(x^2 - 3xy + y^2) = \frac{d}{dx}(1)$$

$$2x - 3(y + x \frac{dy}{dx}) + 2y \frac{dy}{dx} = 0$$

$$(2y - 3x) \frac{dy}{dx} = 3y - 2x$$

$$\therefore \frac{dy}{dx} = \frac{3y - 2x}{2y - 3x}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left[\frac{dy}{dx} \right] = \frac{d}{dx} \left[\frac{3y - 2x}{2y - 3x} \right]$$

$$= \frac{(2y - 3x)(3 \frac{dy}{dx} - 2) - (3y - 2x)(2 \frac{dy}{dx} - 3)}{(2y - 3x)^2}$$

$$= \frac{5y - 5x \frac{dy}{dx}}{(2y - 3x)^2}$$

$$= \frac{5y - 5x \left(\frac{3y - 2x}{2y - 3x} \right)}{(2y - 3x)^2}$$

$$= \frac{5y(2y - 3x) - 5x(3y - 2x)}{(2y - 3x)^3}$$

$$= \frac{10y^2 - 30xy + 10x^2}{(2y - 3x)^3}$$