

判断无穷区间的积分 $\int_2^{\infty} \frac{1}{(x-1)^2} dx$ 是否收敛?

$$= \lim_{t \rightarrow \infty} \int_2^t \frac{1}{(x-1)^2} dx$$

$$= \lim_{t \rightarrow \infty} \int_2^t (x-1)^{-2} dx$$

$$= \lim_{t \rightarrow \infty} - (x-1)^{-1} \Big|_2^t$$

$$= \lim_{t \rightarrow \infty} - \frac{1}{x-1} \Big|_2^t$$

$$= \lim_{t \rightarrow \infty} - \left(\frac{1}{t-1} - \frac{1}{2-1} \right)$$

$$= \lim_{t \rightarrow \infty} - \left(\frac{1}{t-1} - 1 \right)$$

$\infty - 1 = \infty$
 $\frac{1}{\infty} = 0$

$$= 1$$

$\therefore \int_2^{\infty} \frac{1}{(x-1)^2} dx$ 收敛, 而且收敛到 1.