

三角、反三角、雙曲及反雙曲函數的積分

$$\int \frac{\sin x}{1 + \sin x} dx$$

$$= \int \frac{\sin x}{1 + \sin x} \cdot \frac{1 - \sin x}{1 - \sin x} dx = \int \frac{\sin x(1 - \sin x)}{1 - \sin^2 x} dx$$

$$= \int \frac{\sin x(1 - \sin x)}{\cos^2 x} dx = \int \frac{\sin x}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} dx$$

$$= \int \left(\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} - \frac{1 - \cos^2 x}{\cos^2 x} \right) dx$$

$$= \int (\sec x \tan x - \sec^2 x + 1) dx$$

$$= \sec x - \tan x + x + C$$