有理函數積分法.

① 下化 $\dfrac{x^2+x-3}{(x-1)(x^2-x+1)}$ 篇部分分式

② 計算 $\displaystyle\int \dfrac{x^2+x-3}{(x-1)(x^2-x+1)}\,dx$

① $\dfrac{x^2+x-3}{(x-1)(x^2-x+1)} = \dfrac{A}{(x-1)} + \dfrac{Bx+C}{(x^2-x+1)}$

令 $x=1$, $A = \dfrac{1+1-3}{1-1+1} = -1$

乘

同乘 $(x-1)(x^2-x+1)$

$-1(x^2-x+1) + (Bx+C)(x-1)$

∴ $B=2$
$C=2$

$\dfrac{-1}{x-1} + \dfrac{2x+2}{x^2-x+1}$

② $\displaystyle\int \dfrac{-1}{x-1} + \dfrac{2x+2}{x^2-x+1}\,dx$

$= \displaystyle\int \dfrac{-1}{x-1} + \boxed{\dfrac{2x-1}{x^2-x+1}} + \dfrac{3}{x^2-x+1}\,dx$

$\ln|x^2-x+1|$

$= -\ln|x-1| + \ln|x^2-x+1| + \displaystyle\int \dfrac{3}{(x-\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}\,dx$

$= -\ln|x-1| + \ln|x^2-x+1| + 2\sqrt{3}\,\tan^{-1}\dfrac{2(x-\frac{1}{2})}{\sqrt{3}} + C$