

第 6 章 習題簡解

習題 6-1

1. (1) $\frac{1}{8}(2x+1)^4 + c$ (2) $\frac{1}{36}(x^4+1)^9 + c$
- (3) $-\frac{1}{15(x^3+2)^5} + c$ (4) $\frac{1}{8}(1+8x^2)^{\frac{1}{2}} + c$
- (5) $\frac{1}{3}(x^2-1)^{\frac{3}{2}} + c$ (6) $\frac{1}{5}(x+2)^5 - \frac{1}{2}(x+2)^4 + c$
- (7) $\frac{2}{5}(2+x)^{\frac{5}{2}} - \frac{4}{3}(2+x)^{\frac{3}{2}} + c$ (8) $\frac{1}{3}(1+x^2)^{\frac{3}{2}} - (1+x^2)^{\frac{1}{2}} + c$
2. (1) $-\frac{2048}{11}$ (2) 513
- (3) 1 (4) $\frac{1}{3\sqrt{2}}$
- (5) $\frac{8}{3}$ (6) $\frac{506}{75}$
3. Hint: 令 $u = a + b - x$

習題 6-2

1. $\ln|x+4| + c$ 2. $-e^{\frac{1}{x}} + c$
3. $-\frac{1}{6}e^{-3x^2} + c$ 4. $-\frac{1}{6}\ln|1-2x^3| + c$
5. $\ln|e^x - 1| + c$ 6. $x - \ln(1 + e^x) + c$
7. $\ln(e^x + e^{-x}) + c$ 8. $-\frac{1}{e^x + 1} + c$
9. $2\sqrt{e^x - 1} + c$ 10. $2\ln(1 + e^{\sqrt{x}}) + c$
11. $\frac{1}{3}[\ln(1 + e^x)]^3 + c$ 12. $\frac{1}{2}\ln(x^2 + 2x + 3) + c$
13. $2(e^2 - e)$ 14. $\frac{1}{2}(e^5 - e)$

習題 6-3

1. $-\frac{1}{3}\cos(3x+1) + c$ 2. $\frac{1}{3}\sin 3x + c$
3. $\frac{1}{5}\ln|\sec 5x| + c$ 4. $-\frac{2}{3}\cot 3x - \frac{2}{3}\csc 3x - x + c$
5. $x + \cos x + c$ 6. $\frac{1}{5}\ln|3 + \sin 5x| + c$

7. $\frac{2}{3}(5 + \tan x)^{\frac{3}{2}} + c$

8. $\frac{1}{5} \tan 5x + c$

9. $\frac{1}{2} [\ln(\sin x)]^2 + c$

10. $\frac{2}{9}(4 + 3 \sec x)^{\frac{3}{2}} + c$

11. $(\tan^{-1} \sqrt{x})^2 + c$

12. $-2 \cos \sqrt{x} + c$

13. $e^{4+\sin x} + c$

14. $-e^{\cos x} + c$

15. $\ln |e^x - e^{-x}| + c$

16. $\tan^{-1} e^x + c$

17. $\sin(\ln x) + c$

18. $\sin^{-1} \frac{\ln x}{3} + c$

19. $\frac{1}{2}(\sin^{-1} x)^2 + c$

20. $\frac{1}{2} \sec^{-1} x^2 + c$

21. $\frac{1}{3} \sin^{-1} x^3 + c$

22. $\frac{1}{2\sqrt{3}} \tan^{-1} \frac{x^2}{\sqrt{3}} + c$

23. $\frac{1}{2} \cosh^{-1} \frac{2x}{3} + c$

24. $\sinh^{-1} \frac{\ln x}{2} + c$

25. 略

習題 6-4

1. $-x \cos x + \sin x + c$

2. $\frac{1}{3} x^2 \sin 3x + \frac{2}{9} x \cos 3x - \frac{2}{27} \sin 3x + c$

3. $x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x + c$

4. $\frac{1}{3} x^2 e^{3x} - \frac{2}{9} x e^{3x} + \frac{2}{27} e^{3x} + c$

5. $-\frac{\ln x}{4x^4} - \frac{1}{16x^4} + c$

6. $\frac{1}{4} x^4 \ln 5x - \frac{1}{16} x^4 + c$

7. $\frac{1}{2} x^2 \sin^{-1} x + \frac{x\sqrt{1-x^2}}{4} - \frac{1}{4} \sin^{-1} x + c$

8. $x \tan^{-1} x - \frac{1}{2} \ln(1+x^2) + c$

9. $x \ln x - x + c$

10. $x(\ln x)^2 - 2(x \ln x - x) + c$

11. $\frac{1}{2} e^{x^2} (x^2 - 1) + c$

12. $-\frac{1}{2} x^2 e^{-x^2} - \frac{1}{2} e^{-x^2} + c$

13. $\frac{1}{2} e^x (\sin x + \cos x) + c$

14. $\frac{1}{2} (-\csc x \cot x + \ln |\csc x - \cot x|) + c$

15. $x \tan x - \ln |\sec x| + c$

16. $-\frac{x}{4} \sin 2x - \frac{1}{8} \cos 2x + \frac{x^2}{4} + c$

17. $\sin x e^{\sin x} - e^{\sin x} + c$

18. $x \ln(1+x^3) - 3x + \ln|x+1| - \frac{1}{2} \ln(x^2-x+1) + \sqrt{3} \tan^{-1} \frac{2x-1}{\sqrt{3}} + c$

19. $4 - 12e^{-2}$

20. $\frac{1}{2} (\sin^{-1} 1 - 1)$

21. 略 22. 略 23. 略

習題 6-5

1. $-\frac{1}{14}\cos 7x + \frac{1}{6}\cos 3x + c$

2. $-\frac{1}{16}\cos 8x + \frac{1}{4}\cos 2x + c$

3. $\frac{1}{18}\sin 9x + \frac{1}{6}\sin 3x + c$

4. $-\frac{1}{18}\sin(9x+6) + \frac{1}{10}\sin 5x + c$

5. $\frac{\pi}{4}$

6. $\sin x - \frac{1}{3}\sin^3 x + c$

7. $\frac{1}{5}\sin^5 x - \frac{2}{7}\sin^7 x + \frac{1}{9}\sin^9 x + c$

8. $-\frac{2}{5}\cos^{\frac{5}{2}} x + \frac{2}{9}\cos^{\frac{9}{2}} x + c$

9. $\frac{3}{2}\sin^{\frac{2}{3}} x - \frac{3}{8}\sin^{\frac{8}{3}} x + c$

10. $-\frac{2}{3}\cos^{\frac{3}{2}} x + \frac{2}{7}\cos^{\frac{7}{2}} x + c$

11. $\frac{3}{8}x - \frac{1}{4}\sin 2x + \frac{1}{23}\sin 4x + c$

12. $\frac{3}{2}x + \frac{3}{20}\sin 10x + c$

13. $\frac{1}{2}\tan^2 x - \ln|\sec x| + c$

14. $\frac{1}{3}\tan^3 x - \tan x + x + c$

15. $-\frac{2}{3}\cot 3x - \frac{2}{3}\csc 3x - x + c$

16. $\frac{1}{4}\sec^3 x \tan x + \frac{3}{8}\sec x \tan x + \frac{3}{8}\ln|\sec x + \tan x| + c$

17. $\frac{1}{4}\tan^4 x + \frac{1}{6}\tan^6 x + c$

18. $-\frac{1}{12}\cot^4 3x - \frac{1}{18}\cot^6 3x + c$

19. $\csc x + \cot x + c$

20. $\frac{1}{2}\sin 2x + c$

習題 6-6

1. $\frac{x}{4\sqrt{4-x^2}} + c$

2. $\ln\left|\frac{1}{x} - \frac{\sqrt{1-x^2}}{x}\right| + c$

3. $\frac{\pi}{2}$

4. $5\sin^{-1}\left(\frac{x-1}{\sqrt{6}}\right) + c$

5. $\ln(\sqrt{x^2+16}+x) + c$

6. $\frac{x}{9\sqrt{9+4x^2}} + c$

7. $\frac{1}{3}\ln\left|\frac{\sqrt{9+4x^2}-3}{2x}\right| + c$

8. $\ln|\sqrt{x^2+6x+10}+x+3| + c$

9. $\sqrt{4x^2-9} - 3\sec^{-1}\frac{2x}{3} + c$

10. $\frac{\sqrt{x^2-1}}{x} + c$

11. $\ln|x-1+\sqrt{x^2-2x}| + c$

12. $\frac{25}{6} - \frac{25}{8}\ln 3$

13. $\sqrt{2} + \ln(\sqrt{2} + 1)$

14. $\frac{e^x \sqrt{e^{2x} - 9}}{2} - \frac{9}{2} \ln(e^x + \sqrt{e^{2x} - 9}) + c$

15. $\sin^{-1} \frac{x-1}{3} - \sqrt{8+2x-x^2} + c$

16. $\frac{1}{2} \sec^{-1} \left(\frac{e^x}{2} \right) + c$

習題 6-7

1. $\frac{1}{4} \ln \left| \frac{x-2}{x+2} \right| + c$

2. $\frac{2}{5} \ln|x| - \frac{7}{5} \ln|x+5| + c$

3. $-\ln|x-1| + \frac{11}{5} \ln|x-2| - \frac{1}{5} \ln|x+3| + c$

4. $3 \ln|x| - \frac{3}{2} \ln(x^2 + 1) - \tan^{-1} x + c$

5. $\ln|x| + \ln|x-1| + \ln|x+2| + c$

6. $\ln|x+1| + \frac{3}{x+1} - \frac{3}{2(x+1)^2} + c$

7. $\frac{1}{2} x^2 - 2x + \frac{1}{8} \ln|x| + \frac{1}{4x} - \frac{1}{4x^2} + \frac{31}{8} \ln|x+2| + c$

8. $\frac{3}{25} \ln|x| - \frac{1}{25x} - \frac{3}{50} \ln(x^2 + 25) - \frac{1}{125} \tan^{-1} \frac{x}{5} + c$

9. $2 \ln|x-1| - \frac{1}{x-1} - 2 \ln|x+2| + c$

10. $2 \ln|x-1| + \frac{2}{x-1} - \ln(x^2 + 1) + \tan^{-1} x + c$

11. $-\ln|\tan x| + \frac{1}{\tan x} + \ln|\tan x - 1| + c$

12. $-\ln(e^x + 1) + \frac{1}{e^x + 1} + x + c$

13. $-\frac{1}{e^x + 1} + \frac{1}{2(e^x + 1)^2} + c$

14. $\ln|\sin x| - \frac{1}{2} \ln(\sin^2 x + 1) + c$

15. $\frac{1}{\sqrt{2}} \ln \left| \frac{\tan \frac{x}{2} - 1 + \sqrt{2}}{\tan \frac{x}{2} - 1 - \sqrt{2}} \right| + c$

16. $-\tan \frac{x}{2} + x + c$

17. $\frac{1}{5} \ln \left| \frac{2 \tan \frac{x}{2} + 1}{\tan \frac{x}{2} - 2} \right| + c$

18. $\frac{1}{2} \ln \left| \frac{\tan \frac{x}{2} + 1}{\tan \frac{x}{2} - 1} \right| - \frac{1}{\tan \frac{x}{2} + 1} + \frac{1}{(\tan \frac{x}{2} + 1)^2} + c$

習題 6-8

1. (1) 發散 (2) 收斂 (3) 收斂

(4) 發散 (5) 收斂 (6) 收斂

(7) 發散 (8) 發散 (9) 收斂

(10) 發散

2. (1) 收斂 (2) 收斂 (3) 發散

(4) 收斂 (5) 收斂 (6) 發散

習題 6-9

1. 梯形法則 ≈ 3.224 , 辛普森法則 ≈ 3.216
2. 梯形法則 ≈ 5.6724 , 辛普森法則 ≈ 5.6528
3. 梯形法則 ≈ 0.6938 , 辛普森法則 ≈ 0.6932
4. 梯形法則 ≈ 0.6355 , 辛普森法則 ≈ 0.6377
5. 梯形法則 ≈ 1.3695 , 辛普森法則 ≈ 1.3708
6. 梯形法則 ≈ 0.7462 , 辛普森法則 ≈ 0.7468
7. 梯形法則 ≈ 1.1352 , 辛普森法則 ≈ 1.1358