第 9 章 習題簡答

習題 9-1

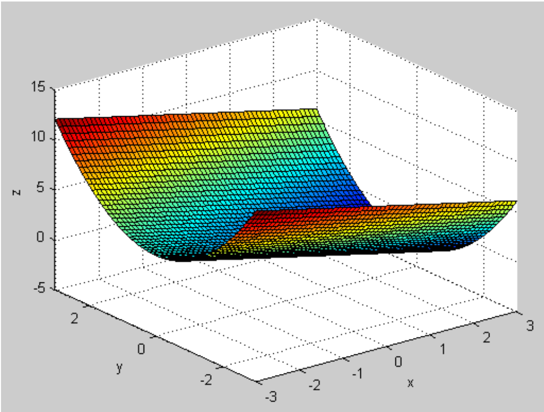
1. (1) $D_f=\Re^2\setminus\{(0,0)\}$ (2) $D_f=\{(x,y)\,|\,x\neq 0, y\neq 0\}$

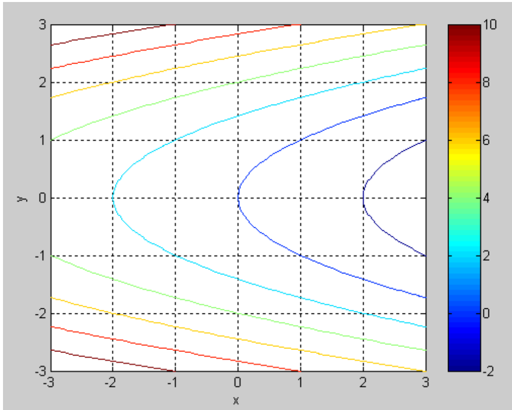
(3) $D_f=\{(x,y)\,\big|\,|x|\ge|y|, x,y\in\Re\}$ (4) $D_f=\{(x,y)\,\big|\,x^2+y^2<9, x,y\in\Re\}$

(5) $D_f=\{(x,y,z)\,\big|\,|x|\neq|z|\}$ (6) $D_f=\{(x,y,z)\,\big|\,x^2+y^2+z^2\ge 5^2\}$

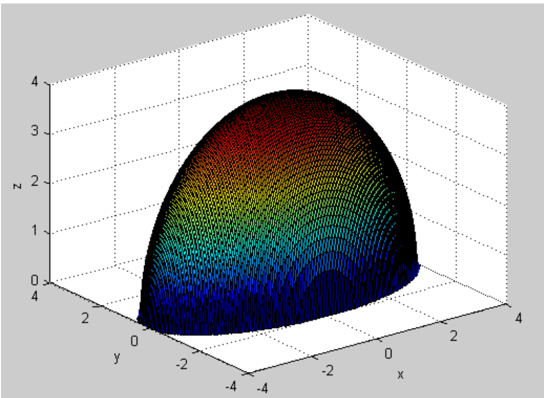
2.(1) $R_f=[10,\infty)$ (2) $R_f=[0,6]$ (3) $R_f=[-1,1]$ (4) $R_f=(-\infty,6]$

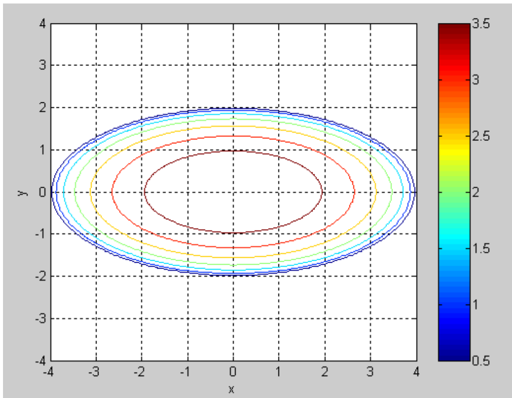
3.(1) $f(x,y)=y^2-x$



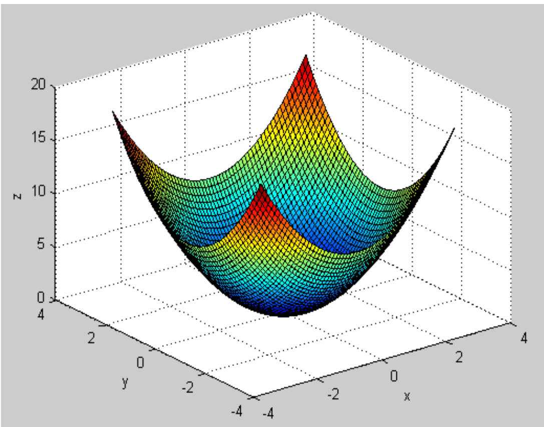


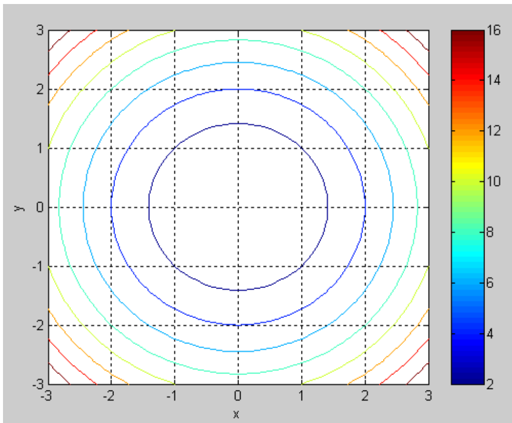
(2) $f(x,y)=\sqrt{16-x^2-4y^2}$



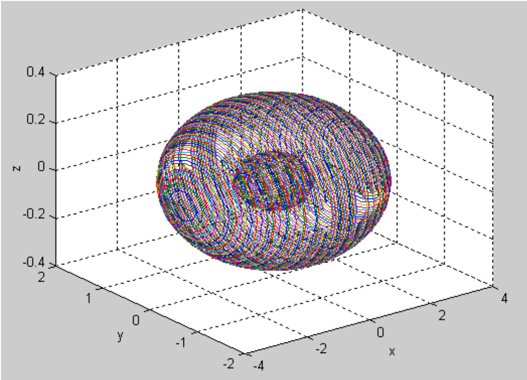


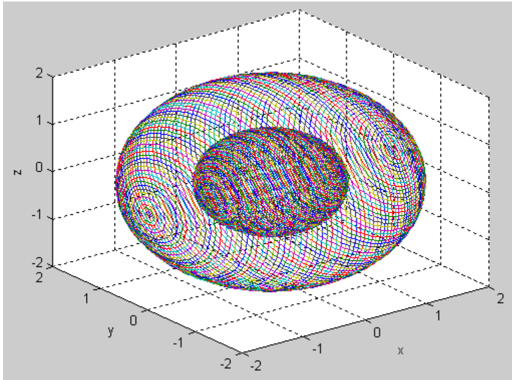
(3) $f(x,y)=x^2+y^2$



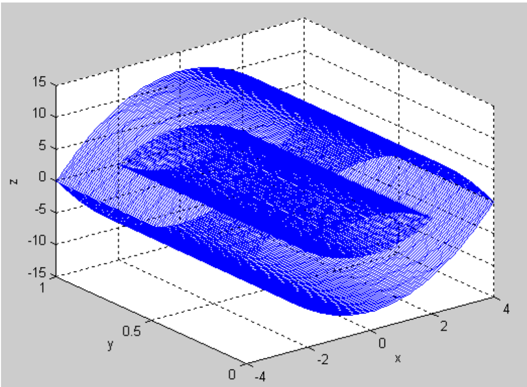


4.(1) $f(x,y,z)=x^2+4y^2+9z^2$ (2) $f(x,y,z)=x^2+y^2+z^2$





(3) $f(x,y,z)=\dfrac{x^2}{4}+\dfrac{z^2}{9}$



習題 9-2

1. $f_x=3x^2+14xy+3,\ f_y=7x^2+24y^2-2$

2. $\dfrac{\partial z}{\partial x}=\tan^{-1}\dfrac{y}{x}-\dfrac{xy}{x^2+y^2},\dfrac{\partial z}{\partial y}=\dfrac{x^2}{x^2+y^2}$

3. $f_x(0,\dfrac{\pi}{4})=-1,\ f_y(0,\dfrac{\pi}{4})=0$ 4.略

5. $\dfrac{\partial f}{\partial z}=(xy)^{\sin z}\cdot\cos z\cdot\ln(xy)$

6. 2 7. 0 8. $\dfrac{\partial R}{\partial R_3}=\left(\dfrac{R_1R_2}{R_2R_3+R_1R_3+R_1R_2}\right)^2$

9. $f_x(0,0)=0,\ f_y(0,0)=0$

10. $f_x(0,0)=1,\ f_y(0,0)=1$

11. $f_{xxy}=\dfrac{6x^2y-2y^3-2yz^2}{(x^2+y^2+z^2)^3},\ f_{xxz}=\dfrac{6x^2z-2z^3-2y^2z}{(x^2+y^2+z^2)^3}$

12. $f_{xx}=6x+4y,\ f_{xy}=4x+4y$

13. $f_{xy}=e^{-xy}(y\sin x-\cos x),\ f_{yx}=f_{xy}$

14. $f_{xy}=ze^{yz}+ze^{xz}+(1+xy)ze^{xy}$ ， $f_{yxz}=(1+yz)e^{yz}+(1+xz)e^{xz}+(1+xy)e^{xy}$ ，

$f_{zxy} = f_{yxz}$

15. 切線斜率：-2，切線方程式：$\begin{cases} z-1=-2(y-2) \\ x=2 \end{cases}$

16. 切線方程式：$\begin{cases} z+3=2(x+1) \\ y=2 \end{cases}$　　17.略　　18.略

習題 9-4

1. $4(3xy+1)t+12(3x^2+1)e^{4t}$
2. $\dfrac{du}{dt}=-\dfrac{1}{t^2}(y+z)+(x+z)e^t-(y+x)e^{-t}$
3. $\dfrac{\partial u}{\partial s}=2xe^t+2yte^s, \dfrac{\partial u}{\partial t}=2xse^t+2ye^s$
4. $\dfrac{\partial u}{\partial s}=2(1+y^2z)s+(z^2+2xyz)t+2(2yz+xy^2)s$，
   $\dfrac{\partial u}{\partial t}=2(1+y^2z)t+(z^2+2xyz)s-2(2yz+xy^2)t$
5. 略　　6. $\dfrac{du}{dx}=\dfrac{y-xe^x}{y^2+x^2}$
7. $\dfrac{du}{dx}=\dfrac{x+y\sin x+xy\cos x+z\sec^2 x}{\sqrt{x^2+y^2+z^2}}$
8. $\dfrac{\partial z}{\partial \theta}\Big|_{r=3,\theta=\pi/6}=\dfrac{1}{2}, \dfrac{\partial z}{\partial r}\Big|_{r=3,\theta=\pi/6}=0$　　9.略　　10. 略
11. 14吋/秒
12. 體積以306呎^3/秒的速度增加。
13. $\dfrac{dy}{dx}=-\dfrac{2x+3y}{3x-4y}$
14. $-\dfrac{2xe^{3y}}{3(x^2e^{3y}+y^2)}$
15. $\dfrac{\partial z}{\partial x}\Big|_{(0,0,2)}=0, \dfrac{\partial z}{\partial y}\Big|_{(0,0,2)}=\dfrac{1}{3}$
16. $\dfrac{\partial z}{\partial x}=-\dfrac{e^{yz}+yze^{xy}}{xye^{yz}+xye^{xz}}, \dfrac{\partial z}{\partial y}=-\dfrac{xze^{yz}+e^{xz}-2y}{xye^{yz}+xye^{xz}}$
17. $\dfrac{dy}{dx}=\dfrac{-(x+2y+z)}{z-y}, \dfrac{dz}{dx}=\dfrac{x+y+2z}{z-y}$
18. $\dfrac{dy}{dx}=\dfrac{-xy^2-2x+z}{2x^2y+2y}, \dfrac{dz}{dx}=\dfrac{-(2xyz+2y^3+4y)}{2x^2y+2y}$

習題 9-5

1. $\Delta f=0.508, df=0.5$　　2. $\Delta z=-0.87, dz=-0.9$
3. (1) $dz=\dfrac{3x^2dx-4ydy}{\sqrt{2x^3-4y^2}}$　　(2) $dz=\dfrac{ydx-xdy}{|y|\sqrt{y^2-x^2}}$

(3) $dz = (3+2x^2)x^2e^{x^2-y^2}dx - 2x^3ye^{x^2-y^2}dy$

(4) $du = z\cos(y+z)dy + [\sin(y+z) + z\cos(y+z)]dz$

(5) $du = \dfrac{-xzdx - yzdy + (x^2+y^2)dz}{(x^2+y^2+z^2)\sqrt{x^2+y^2}}$

(6) $du = [\dfrac{1}{x} - z\sin(xz)]dx + [\dfrac{1}{y} + ze^{yz}]dy + [ye^{yz} - x\sin(xz)]dz$

4. ≈ 10.004　　5. ≈ 0.41119

6. 0.11 呎^2　　7. 0.07854 立方公尺

8. $\dfrac{1}{T}(0.3\%) - \dfrac{1}{V}0.8\%$　　9. 最大誤差：-0.0002，百分誤差：-0.00167%

習題 9-6

1.(1) $\nabla f = \langle 3x^2 - 2xy^2, -2x^2y + 3y^2 \rangle$　(2) $\nabla f = \langle -4xy^3e^{-2x^2y^3}, 1 - 6x^2y^2e^{-2x^2y^3} \rangle$

(3) $\nabla f = \langle \dfrac{2xy^3}{z^4}, \dfrac{3x^2y^2}{z^4}, \dfrac{-4x^2y^3}{z^5} \rangle$　(4) $\nabla f = \langle \dfrac{x}{\sqrt{x^2+y^2z^2}}, \dfrac{yz^2}{\sqrt{x^2+y^2z^2}}, \dfrac{y^2z}{\sqrt{x^2+y^2z^2}} \rangle$

2. $\langle -12, -9 \rangle$　　3. $\langle \dfrac{4}{25}, \dfrac{-3}{25}, \dfrac{1}{5} \rangle$

4. $\dfrac{136}{13}$　　5. $\sqrt{409}$

7. 方向應指向 $\langle 5,5 \rangle$，此時方向導數的最大值為 $5\sqrt{2}$

8. $\dfrac{1}{3}$　　9. $\sqrt{41}$　　10. $\dfrac{-7}{\sqrt{17}}$

習題 9-7

1. 切平面方程式：$2x - y + z - 9 = 0$，法線：$\dfrac{x-2}{2} = \dfrac{y+1}{-1} = \dfrac{z-4}{1}$

2. 切平面方程式：$4x + 5y - 2z - 16 = 0$，法線：$\dfrac{x-1}{4} = \dfrac{y-2}{5} = \dfrac{z+1}{-2}$

3. 切平面方程式：$4x - 8y - z - 8 = 0$，法線：$\dfrac{x-2}{4} = \dfrac{y+1}{-8} = \dfrac{z-8}{-1}$

4. 切平面方程式：$8x - 5y - 6z = 0$，法線：$\dfrac{x-1}{8} = \dfrac{y+2}{-5} = \dfrac{z-3}{-6}$

5. 法平面方程式：$x + 2y + 3z - 6 = 0$， 切線參數式：$\begin{cases} x = 1+t \\ y = 1+2t, t \in \Re \\ z = 1+3t \end{cases}$

6. 法平面方程式：$4y + z = 0$，切線參數式：$\begin{cases} x = 3 \\ y = 4t, t \in \Re \\ z = t \end{cases}$

7. 略　　8. 略　　9. 略　　10. 切線：$\begin{cases} x = 4 - t \\ y = -1 \\ z = t \end{cases}, t \in \Re$

習題 9-8

1.(1) 相對極小值：$f(1,2) = -2$

(2) 鞍點：$(\sqrt{\frac{3}{2}}, \sqrt{6}, 3\sqrt{6})$ ，$(-\sqrt{\frac{3}{2}}, -\sqrt{6}, -3\sqrt{6})$ ，$(1,1,0), (-1,-1,0)$

(3) 相對極大值：$f(\frac{2}{3}, \frac{4}{3}) = \frac{59}{27}$ ，鞍點：$(0,0,1), (0,4,1), (2,0,1)$

(4) 鞍點：$(0,0,0)$,相對極小值：$f(1,1) = -1$

(5) 相對極大值：$f(0,0) = 1$，相對極小值：$f(\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}) = \frac{1}{2}$ ，

鞍點：$(0, \pm\frac{1}{\sqrt{2}}, \frac{3}{4}), (\pm\frac{1}{\sqrt{2}}, 0, \frac{3}{4})$

2. $(0,0,\pm 2)$

3. 當長、寬、高均為 $\sqrt{\frac{A}{6}}$ 時有最大體積 $\frac{A\sqrt{6A}}{36}$

4. 長：寬：高 $= 2:2:1$

5. 當 $x = y = \frac{12}{5}, z = \frac{6}{5}$ 時有最大值 $\frac{124416}{3125}$

6. 當 $x = 6, y = 2, z = 4$ 時有極大值 48

7. $(x, y, z) = (-\sqrt{65}, -\sqrt{65} - 8, 0)$

8. 無解