On the Truth Value of Null Statements*

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This paper primarily deals with the truth value of null statements. Based on Russell (1905), a null statement such as *The King of France is bald* is analyzed as false with the assumption of entailments; on the other hand, Frege (1892) argues that a null statement does not have any truth value with the assumption of presuppositions. However, in my talk, I propose a mathematical view of null statements and that a null statement is always true, different from Russell's and Frege's analyses.

Keywords: truth value, null statement

1. Introduction

A null statement is a statement that contains an argument that does not really exist, such as *The King of France is bald* (Russell 1905) (Since there is no king in France nowadays, this statement is a null statement.) In the study of logic, judging the truth value of null statements has been a great and controversial topic, widely discussed by logicians, mathematicians, and semanticists. One of the most popular ideas is proposed by Frege (1892), who supposes that a null statement does not have any truth value at all; another is proposed by Russell (1905), who claims that a null statement is always false. These two proposals seem not to contradict each other. However, if the truth value of a null statement is examined by mathematical

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induction, the result is that a null statement is always true. This result from mathematical induction clearly contradicts Russell’s proposal because logically a statement cannot simultaneously be true and false.

In this paper, I will argue that Russell’s and Frege’s analyses need further consideration with the view of mathematics, and that a null statement is always true, as suggested by the result from the mathematical induction, which is logically preferred.

For the study of semantics through history, mathematics and logic have been crucial tools, especially on the issues originally raised by the field of logic, such as judging the truth value. Therefore, when examining the truth value, mathematics cannot be ignored even though semantics and mathematics are different academic disciplines. As a result, the methodology of this paper is based on the view of mathematics.

In Section 2, I will examine Russell’s and Frege’s proposals, and I will demonstrate that both Russell’s and Frege’s proposals do not fit the mathematical view. Also, I will introduce the details of the result of mathematical induction. Finally, I will conclude my ideas in Section 3.

2. The Viewpoint of Mathematics

2.1 Russell’s Argument

Russell (1905) argues that the truth value of a null statement, such as *The King of France is bald* (Russell 1905), is false[1]. The primary idea in his proposal is that a statement must have

[1] Russell’s example for null statements *The King of France is bald* is actually not an ideal example because it would not be a null statement if the time were, for instance, the 17th century, during which there were a king of France. Here, I still use Russell’s example through my paper since it is a classic example of null statements, but I will also use my own example stated in (3): *The triangle with two right angles is never found*, which is more ideal than Russell’s since a triangle always cannot have two right angles during any time.

certain entailments, as shown in (1).

(1) *The King of France is bald* must entails that (i) there is a king of France, (ii) there is one and only one king of France, and (iii) the individual is bald.

The entailments can be stated in semantic logical forms, as in (2).

(2) $\exists x(\text{king'}(x) \wedge \text{French'}(x) \wedge \sim\exists y((y \neq x) \wedge \text{king'}(y) \wedge \text{French'}(y)) \rightarrow \text{bald'}(x))$

Since the entailments (i) and (ii) are false, the null statement is false. This concept of entailments appears to be reasonable, but the illogical point is that (1) is Russell's (1905) assumption, which is not logically proved through any mathematical methods. Therefore, Russell's idea of entailments does not fit in the logic in mathematics. This fact can be clearly illustrated by another null statement in (3):

(3) The triangle with two right angles is never found.

According to the knowledge of geometry, it is impossible for a triangle to have two right angles, so (3) is a null statement. Here, based on Russell's assumption in (1), (3) is false since (i) and (ii) are false. However, (3) is logically true due to the fact that *the triangle with two right angles* is the empty set, which includes only the empty set as its subset so it cannot be found and nothing can be found in it (Zermelo 1908). Obviously, Russell's assumption in (1) needs further consideration.

Despite the illogical idea in (1), Russell's viewpoint seems to be supported by the analysis based on the idea of regarding *The King of France is bald* as a subset of a bigger set *The King of France*, as illustrated in (4).

(4) {The King of France} = $\emptyset$

{The King of France is bald} $\subseteq$ {The King of France}

$\because$ $\emptyset = \{x: x = \emptyset\}$

$\therefore$ {The King of France is bald} $= \emptyset$ (false)

⇨ *The King of France is bald* is false. #

However, this supporting idea from set theory may be rejected if it is examined more carefully, as shown in (5).

(5) Let D be a set of all the individuals that exist in the real world.

According to the common knowledge in linguistics, English is a Germanic language rather than a Romance language.

a. $\{x \in D$: English is a Germanic language$\} = D$ (true)

b. $\{x \in D$: English is a Romance language$\} = \emptyset$ (false)

The reason for (5b) to be false is actually different from that in (4). In (5b), the truth value is false because English is NOT a Romance language. In contrast, in (4), *The King of France is bald* is concluded as false NOT BECAUSE *The King of France is not bald* is true. (It should be noticed that *The King of France is not bald* is also false in Russell's analysis.) Therefore, based on the analysis of mathematical sets, the conclusion of (4) cannot be that *The King of France is bald* is false although the reached result is {The King of France is bald} $= \emptyset$.

2.2 Frege's Argument

Frege (1892) points out that a statement must presuppose that the elements in the statement must exist in the real world. Based on Frege’s idea and the notion that a statement has its truth value if and only if its presupposition is true, *The King of France is bald* does not have any truth value since the presupposition *There is a King of France* is false.

However, there can be a serious problem is Frege’s analysis if the null statement in (3) is considered. Since there is no triangle with two right angles, the presupposition of (3) *There is a triangle with two right angles* is false, and therefore, (3) does not have any truth value in Frege’s analysis. Due to the fact that (3) is logically true, Frege’s proposal is also rejected.

2.3 The Result from Mathematical Induction

If *The King of France is bald* is analyzed with mathematical induction, as in (6), the result is that *The King of France is bald* is true.

(6) Proof:

The King of France is bald is the prime statement, and it can have a corresponding composite statement *If he is the King of France, he is bald*.

a. The King of France is bald (*prime statement*)

b. If he is the King of France, he is bald (*composite statement*)

First of all, the semantic equivalence between (6a) and (6b) is proved in the following procedure:

a. $\exists x(\text{king'}(x) \wedge \text{French'}(x) \rightarrow \text{bald'}(x)) = A$

b. $\exists x(\text{king'}(x) \wedge \text{French'}(x)) \rightarrow \exists x(\text{bald'}(x))$

$= \exists x(\text{king'}(x) \wedge \text{French'}(x) \rightarrow \text{bald'}(x)) = B$

$\because\ \exists x(\text{king'}(x) \wedge \text{French'}(x)) \rightarrow \sim\exists y((y \neq x) \wedge \text{king'}(y) \wedge \text{French'}(y))$

$\therefore\ \exists x(\text{king'}(x) \wedge \text{French'}(x) \rightarrow \text{bald'}(x))$

$= \exists x(\text{king'}(x) \wedge \text{French'}(x) \wedge \sim\exists y((y \neq x) \wedge \text{king'}(y) \wedge \text{French'}(y)) \rightarrow \text{bald'}(x))$

$= \exists x(\text{king'}(x) \wedge \text{French'}(x) \rightarrow \text{bald'}(x))$

⇨ B = A, (6b) is semantically equivalent to (6a). #

Since (6a) and (6b) are semantically equivalent, (6b) can be used to examine the truth value of (6a).

Let the statement *If P, then Q* represents (6b). Based on Wittgenstein's (1921) truth table of logical implication, as in Table 1, since *P*, *he is the King of France*, is false, no matter what truth value *Q* has, *If P, then Q* is true.

Table 1

Logical Implication (Wittgenstein 1921)

P	***Q***	***P* → *Q***
T	T	T
T	F	F
F	T	T
F	F	T

(6b) is true → (6a) is true, given that *The King of France is bald* is true. #

The idea indicated by (6) is that a null statement is always true. Here, a question may be raised: Is *The triangle with two right angles is found* also true? According to the analysis in (6), *The triangle with two right angles is found* is true, but obviously the reality tells that there is no triangle with two right angles, so it appears that the result of mathematical induction has a problem. However, this problem can be solved if there is a more detailed examination on a null statement that is involved in the question whether or not there is an inexistent argument in the real world, such as *The triangle with two right angles is found / never found*.

(7) Proof:

The triangle with two right angles is found can be written in details as *The triangle with two right angles is found in Jessica's lab of mathematics*, as in (7a) and (7b).

a. *The triangle with two right angles is found.*

b. *The triangle with two right angles is found in Jessica's lab of mathematics*.

(7b) should be considered true. The logic here is that NOTHING (an inexistent thing) can be found everywhere. This idea may be intuitively awkward, but logically reasonable if (7b) is represented by sets. *The triangle with two right angles* is the empty set, and *Jessica's lab of mathematics* is a set including finite elements, such as assistants, computers, books, and so forth, as in (7c) and (7d).

c. *The triangle with two right angles* = Ø

d. *Jessica's lab of mathematics* = {assistants, computers, books, …}

According to the property of the empty set, the empty set is included in every given set

(Let S be any set. ∀S: $\emptyset \subseteq S$), so (7c) is included in (7d). As a result, the set *Jessica's lab of mathematics* can be represented as (7e).

e. *Jessica's lab of mathematics* = {assistants, computers, books, the triangle with two right angles (∅), …}

Based on (7e), *The triangle with two right angles is found in Jessica's lab of mathematics* is true. #

The mathematical induction indicates that a null statement is always true. This result can also be an argument against Russell's proposal. Since a statement cannot be true and false as the same time, Russell's idea assigning a null statement the value of FALSITY does not fit the mathematical view.

3. Conclusion

In my talk, both Russell's and Frege's arguments are rejected. The truth value of null statements should not be false or without any truth value. In fact, a null statement is always true based on the mathematical induction in (6) and (7).

For future research, intentional semantics can be taken into consideration in order to make such study with a wider view.

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論空敘述之真假值

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本論文主題是論證空敘述的真假值。羅素 (Russell 1905) 提出敘述必有其引出值，指出空敘述應為假；另一方面，福瑞格 (Frege 1892) 提出敘述應有蘊含值，認為空敘述其實並沒有真假值。然而，我的論點與羅素和福瑞格不同，我以數學分析方法證明空敘述應為真。

關鍵詞：真假值、空敘述